THEORETICAL AND EXPERIMENTAL STUDIES OF THE NATURE AND CHARACTERISTICS OF SPACE-RELATED PLASMA RESONANCE PHENOMENA

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FOREWORD

The subjects of this research grant are theoretical and experimental studies of the nature and characteristics of space-related plasma resonance phenomena. These studies have been proceeding under the direction of Prof. F. W. Crawford since the award of the grant on 1 May 1965. The current funding period is for twelve months, from 1 July 1969 to 30 June 1970. This is the ninth semiannual progress report on the work, and covers the six-month period from 1 July to 31 December 1969.

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I. INTRODUCTION

The theoretical and experimental work carried out during the initial stages of this NASA grant was concerned with the response of the ionosphere to RF pulses. Its general intent was to elucidate the mechanisms of plasma pulse response phenomena observed by space-probing vehicles such as rockets and satellites. Interest in this area had been stimulated by results obtained from 1963 onwards by the Canadian topside sounder satellite, "Alouette I." [1-7] These indicated that shock excitation of the ionosphere by a pulsed variable frequency signal, from a transmitter carried by the satellite, was followed by prolonged ringing whenever the frequency coincided with a harmonic of the local electron cyclotron frequency, or with the local upper hybrid frequency.

Although the origin of these resonances was puzzling at first, it was soon established[8,9] that they could be well understood in the light of warm magnetoplasma theory and in particular in terms of the electrostatic modes known as cyclotron harmonic waves (CHW).[10,11]

The initial aims of our research program were to study the dispersion characteristics of these waves, both theoretically and experimentally in laboratory plasmas, and to reproduce the Alouette ringing phenomenon. This was done successfully for propagation perpendicular to the magnetic field.[12,13] Since the agreement between theory and experiment was very good, the possibility arose of using CHW as the basis of plasma diagnostic techniques. Several approaches to this problem have been, or are being, studied under this grant. They include resonance rectification, pulse transmission, and impedance measurement. It is hoped that this work will point the way to future space plasma experiments involving Alouette resonances, and that the diagnostic techniques developed may be of use in the measurement of such quantities as electron density and temperature, and local magnetic field strength.

Plasma wave propagation and resonance phenomena of the types just mentioned do not necessarily require electrode structures to excite them: If the plasma electron velocity distribution is non-Maxwellian, e.g., due to the presence of a group of fast electrons interacting with a background Maxwellian plasma, CHW amplification and noise emission may result.[14] We have been examining such possibilities, both experimentally and theoretically,[15] with a view to determining whether they might occur in space plasmas where energetic electrons are known to be present. Two examples of such situations are the auroral zones and the Earth's bow shock.

As a complement to the work on plasma pulse response phenomena such as might be observed by space-probing vehicles, attention has been given under this grant to pulse response phenomena observed by ground-based transmitter/receiver systems, and part of our program has dealt with the existence and mechanism of very long delayed radio echoes. This phenomenon was first observed in the late 1920's and early 1930's,[16] and manifested itself as reception of Morse signals with delays of up to tens of seconds after their transmission. An ionospheric sounding system was run with the object of obtaining evidence for the occurrence of long delayed echoes (LDE). This work was terminated at the end of last contracting period, and transferred to NSF sponsorship.

A further project concerned with resonant interactions observable from the ground has been taken up during the reporting period. This is concerned with nonlinear interaction of two signals at frequencies ω_1, ω_2 to give a radiated signal at frequency $\omega_1 \pm \omega_2$. Experimental observation of such interactions has been reported for meteor trails in auroral zones[17] and from plasma columns in the laboratory.[18] We have carried out some preliminary theory to determine the approximate strength of the interaction, and intend to continue with experiments to demonstrate it.

Detailed comments on the progress made during the reporting period are given in Sections II and III, and our future program is outlined in Section IV.

II. CYCLOTRON HARMONIC WAVE STUDIES

Our present work follows two lines: first, the amplification of CHW by electron beams, and second their excitation by suitable antennas. Recently, most of our emphasis has been on the first. In the last semi-annual report, some theory for CHW amplification was given[15] which has since been accepted for publication. This work is regarded as complete pending further experimental work which may demand further analysis and computation. During the reporting period, it has been possible to acquire a complete apparatus for beam/plasma interaction studies, and modifications to it have been proceeding. Concurrent with these, some theory has been carried out to determine how longitudinal spreading in beam energy occurs when, for example, a group of almost monoenergetic electrons is injected into an auroral zone and interacts with the background plasma. This spreading is, of course, independent of any transverse spreading that may occur due to transfer of transverse beam energy to CHW. It is, however, strongly relevant to any laboratory experiment in which amplification of CHW and the plasma frequency will be competitive unless the longitudinal beam energy spread is significant.

(A) Wave Excitation by an Electron Beam

(a) Theoretical Work

If the energy transfer from the beam to the plasma is small, it is permissible to use a nonadiabatic approach in describing the process of beam/plasma interaction. Under such circumstances, the plasma is a passive background determining the dispersion of the unstable wave. The beam particles interact with the wave fields, and it may be assumed that the latter are not altered appreciably due to the presence ofthe beam. Thus, weak interaction implies that the parameters characterizing the beam particles change only very slowly with respect to the wave period, or alternatively, that the energy loss from the beam is small compared to the initial energy.

Restricting ourselves to a one-dimensional treatment, the interaction is fully described by the equation of motion of a beam particle in the electric field of a traveling wave.

Let the wave be

$$E(z,t) = E_0 \cos(\omega t - kz + \varphi) \quad . \tag{1}$$

We then have for the equation of motion,

$$\frac{dv_b(z,t)}{dt} = \frac{-eE_0}{m} \cos(\omega t - kz + \varphi) \quad , \tag{2}$$

to describe the change in particle velocity, $v_b(z,t)$. The phase, φ, gives the phase of the wave at the instant, and at the position, where the beam particle is injected into the wave. For convenience, we assume this to be at $z = 0$ and $t = 0$. Further, we assume that particles are injected with a fixed velocity, v_0, so that we have the initial condition

$$v_b(0,0) = v_0 \quad . \tag{3}$$

In a laboratory experiment, the beam emerges from a gun at $z = 0$, travels through the plasma, and reaches a boundary at $z = z_0$. If a velocity analyzer is located at z_0, the question arises of what will be measured. In typical experimental setups for CHW studies, the frequencies are in the GHz range, so a velocity analyzer will not be able to measure the instantaneous beam velocity distribution function, $F(z_0,v,t)$. It will give the average over the period of the unstable wave,

$$\langle F(z_0,v,t)\rangle = \frac{1}{T}\int_0^T F(z_0,v,t)dt \quad , \tag{4}$$

assuming that the analyzer can follow the slower changes in $\langle F\rangle$ due to growth of the amplitude of an absolutely unstable wave. If the wave is convectively unstable, $\langle F\rangle$ can be time independent.

Even though the beam electrons are extracted from a gun, this does not imply that the beam particles all have the same energy. As particles are injected into an existing wave, they acquire potential energy at entry, with a magnitude depending on the phase, φ. In an experimental situation, it follows then that particles are injected with variable energy. From Eq. (1), the wave potential at the injection point is

$$V(0,t) = \frac{E_0}{k} \sin \varphi \quad , \tag{5}$$

and the total injection energy of the beam particles is

$$U(\varphi) = U_{kin} + U_{pot} = \frac{1}{2} mv_0^2 - \frac{eE_0}{k} \sin \varphi \quad . \tag{6}$$

A very elementary calculation of $\langle F \rangle$, neglecting the variation in injection energy, Eq. (6), shows that

$$\langle F \rangle \propto \left[\left(\frac{eE_0}{m(\omega - kv_0)} \right)^2 - (v - v_0)^2 \right]^{-1/2} \quad .$$

Measurements,[19] however, have given a rectangular shape for $\langle F \rangle$. The aim of the present calculations is to find out which of two possible effects that arise in this nonadiabatic treatment is the more important in distorting the shape of $\langle F \rangle$: nonlinearity in the equation of motion (Eq. (2)), or the spread in initial injection energy. In both cases, the distortion of $\langle F \rangle$ is the result of phase-mixing, and the distinction between them makes sense only when the particles are not trapped in the wave. All trapped particles move with the same average velocity, equal to the wave phase velocity, $v_f (= \omega/k)$. Phase-mixing is thus only possible through the nonlinearity in Eq. (2). For untrapped particles, it is possible to use linearized equations, but retaining phase-mixing by taking account of Eq. (6). Phase-mixing results from a difference in average velocity.

Division into trapped and untrapped particles is made by use of Eq. (2). The equation of motion has a first integral, which may be written, after applying the initial condition of Eq. (3), as

$$[(v_f - v_b(z,t)]^2 - (v_0 - v_f)^2 = 2\hat{v}(v_0 - v_f)[\sin(\omega t - kz + \varphi) - \sin \varphi] \quad , \tag{7}$$

where $\hat{v} = eE_0/mk(v_0 - v_f)$. A condition for beam/plasma interaction is that $v_0 > v_f$, and hence $\hat{v} \geq 0$. The trapping limit is found from the condition that the inflection point, where the particle velocity in the wave frame changes sign, so that $v_b(z,t) = v_f$, lies on the top of the potential hill. For this condition, Eq. (7) gives

$$\sin \varphi_c = \frac{v_0 - v_f}{2\hat{v}} - 1 \quad . \tag{8}$$

Trapping occurs for

$$\varphi_c < \varphi < \pi - \varphi_c \quad , \tag{9}$$

provided that φ_c exists, or that $(v_0 - v_f) < 4\hat{v}$. Both Eqs. (7) and (8) are exact.

In the following, we will restrict ourselves to untrapped particles and work out the particle velocity. Equation (7) gives $v_b(z,t)$ directly, but integrating a second time results in an elliptic integral. Computer solutions for this have been published,[20] but did not distinguish between the causes of phase-mixing mentioned above.

Linearizing Eq. (7) gives

$$v_b(z,t) = v_0 + \hat{v}[\sin(\omega t - kz + \varphi) - \sin \varphi] \quad , \tag{10}$$

valid when $v_0 - v_f \gg |v_b - v_0| = \hat{v}$. On integrating Eq. (10) with respect to time, we obtain the following integral and its solution

$$t = \int_0^{\omega t - kz_0} \frac{d\psi}{\omega - kv_0 + k\hat{v} \sin \varphi - k\hat{v} \sin(\psi + \varphi)} \quad ,$$

$$\tan\left[\frac{at}{2}\left[1 - \frac{k\hat{v}}{b}\left(\sin \varphi + \tan\left(\frac{\omega t - kz_0}{2}\right) \cos \varphi\right)\right]\right] = \frac{a}{b} \tan\left(\frac{\omega t - kz_0}{2}\right) \quad , \tag{11}$$

$$a^2 = b^2 - k^2\hat{v}^2 \quad , \quad b = \omega - kv_0 + k\hat{v} \sin \varphi \quad ,$$

where the constants a, b are functions of the initial phase, φ, only. Equation (11) gives the time needed by a beam particle to traverse the plasma, or to cover a distance z_0, as function of the phase, φ. In this derivation, the equation of motion was linearized.

Due to the term $\hat{v} \sin \varphi$ in Eq. (10), particles have different average velocities. This is caused by the spread in initial energy, and is, therefore, the effect of phase-mixing on the distribution function. The plot of phase trajectories shown in Fig. 1 illustrates this point. The trajectories are given by Eq. (10), considering only the spatial dependence. At $z = 0$ all particles start with the same velocity, v_0. For $\varphi = \pi/2$, the particle has the minimum initial energy. For $\varphi = 3\pi/2$, it has the maximum initial energy. When particles start at the same time, $t = 0$ at $z = 0$, but with different phases, they have reached different positions (indicated by dots) at the time for which Fig. 1 is drawn. Conversely, particles with $\varphi = \pi/2$ need a longer time to reach point z_0, and arrive with a different velocity, although they started with the same velocity, v_0.

Knowing $t(z)$ (Eq. (11)) and $v_b(z,t(z))$ (Eq. (10)), we can obtain the distribution function. The particles leave the gun with a fixed velocity. So the distribution function there is just a δ-function, $F(0,v,0) = n_0 \delta(v - v_0)$. At the collector side, the distribution will be

$$F(z_0,v,t) = n(z_0,t)\ \delta(v - v_b(z,t)) \quad , \tag{12}$$

with the density determined by the continuity equation

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}(n v_b) = 0 \quad . \tag{13}$$

Linearizing Eq. (13), we obtain

$$n(z,t) = n_0 \left[1 + \frac{k\hat{v}}{\omega - kv_0} \sin(\omega t - kz + \varphi) \right] \quad . \tag{14}$$

Equations (10), (11) and (14) provide all necessary information to compute the distribution function $F(z_0,v,t)$ at any arbitrary position z_0. The time-averaged distribution is obtained by averaging F (Eq. (12)) over the initial phase. With injection of a smooth beam, phase and time in the frame of an observer watching the beam are identical at $z = 0$, and

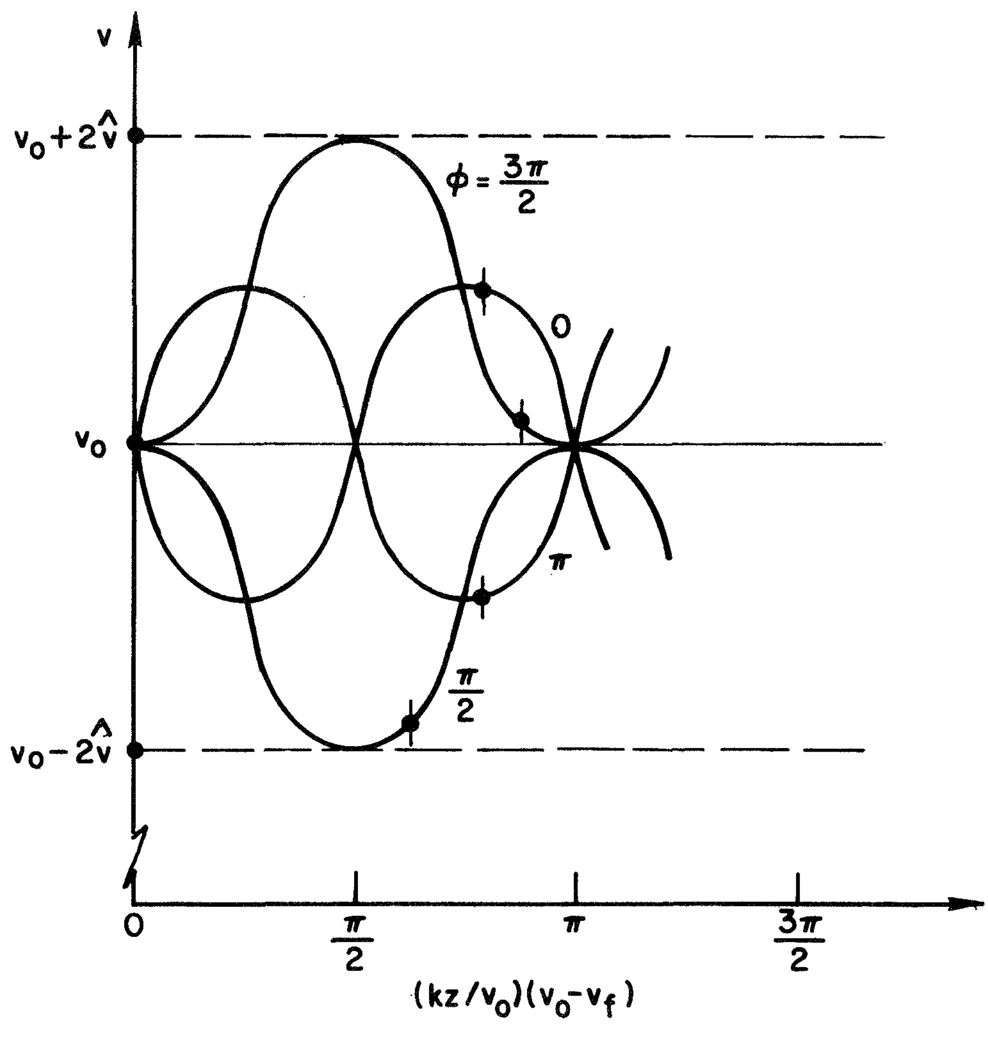

FIG. 1. VARIATION OF PARTICLE VELOCITY WITH DISTANCE AT FIXED TIME.

can be interchanged. At an arbitrary position, $z = z_0$, we have to take care to extend the integration over a large enough number of phase periods: slow particles from one period may be overtaken by fast particles from a later period. For the averaged distribution, we have

$$\langle F \rangle = \frac{1}{\ell\pi} \int_0^{\ell\pi} n(z_0,t)\ \delta(v - v_b(z_0,t))d\varphi$$

$$= \frac{n_0}{\ell\pi} \int_0^{\ell\pi} \frac{\left[1 + \frac{k}{\omega - kv_0}(v_b - v_0 + \hat{v}\sin\varphi)\right]\ \delta(v - v_b)dv_b}{\frac{\partial v_b}{\partial\varphi} - \frac{\partial v_b}{\partial t}\frac{dt}{d\varphi}} \quad . \quad (15)$$

Equation (15) constitutes an analytic expression for the beam distribution function at any point, z_0. It was obtained by using the linearized equations of motion and of continuity, together with the initial condition, $v_b = v_0$. Evaluation of $\langle F \rangle$ is possible, however, only by numerical methods, unless approximations are made.

Suppose $(v_0 - v_t) \gg \hat{v}$. Then Eq. (11) becomes simply

$$t = \frac{z}{v_0} \quad , \qquad (16)$$

which amounts to neglecting the initial variation in energy of the beam particles. Using Eq. (16) in all subsequent expressions, we obtain

$$\langle F \rangle = \frac{n_0}{\pi}\left\{\frac{1 + \frac{v - v_0}{2(v_0 - v_f)}}{\left[4\hat{v}^2 \sin^2\alpha - (v - v_0)^2\right]^{1/2}} + \frac{|\cot\alpha|}{2(v_0 - v_f)}\right\} \quad ,$$

$$\text{for } |v - v_0| \le 2\hat{v}\sin\alpha \quad ,$$

$$= 0 \qquad \text{for } |v - v_0| > 2\hat{v}\sin\alpha \quad , \qquad (17)$$

where $\alpha = (kz/2)(1 - v_f/v_0)$. The asymmetry in $\langle F \rangle$ is due to the dependence of n on $v_b(z,t)$. The asymmetry has been greatly exaggerated in Fig. 2.

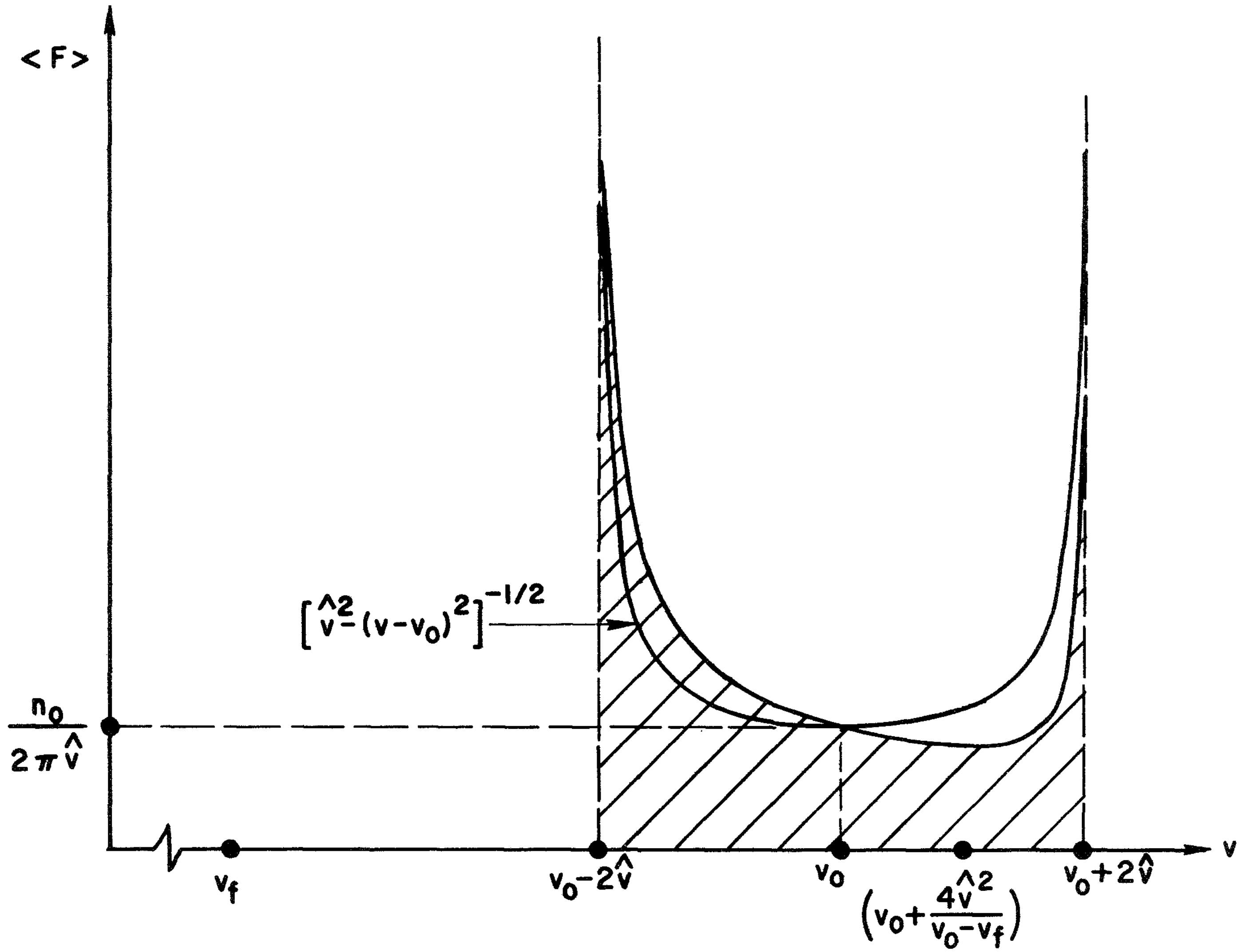

FIG. 2. TIME-AVERAGED DISTRIBUTION FUNCTION, $\langle F\rangle$, AT A POSITION WHERE IT ATTAINS ITS MAXIMUM WIDTH $[z = (p + 1/2)v_0/(v_0 - v_f),\ p = 0,1,2..]$.

As a check on the correctness of Eq. (17), it is easily shown that

$$\int_{z}^{z+\lambda} dz \int_{-\infty}^{\infty} dv \ \langle F \rangle dv = n_0 \quad . \tag{18}$$

It is also worthwhile to note that Eq. (17) holds exactly when $\langle F \rangle$ is calculated in the wave frame, using the full nonlinear equation of motion.

When the condition $v_0 - v_f \gg \hat{v}$ is relaxed, calculations become very complicated. Concentrating on particles with initial phase 0 or π, it is possible, however, to show that the minimum in $\langle F \rangle$, near $v = v_0$, tends to fill up. This flattening of the distribution function, measured experimentally, becomes important for a distance

$$z \approx \pi \frac{v_0^2}{\hat{v}(v_0 - v_f)} \quad . \tag{19}$$

(b) Experimental Work

The apparatus to be used in our experimental studies of beam-stimulated CHW was taken over from a project terminated under another contract. It is described in detail in Ref. 21, and is shown schematically in Fig. 3. A few modifications have been made to it during the reporting period, to bring it to a form suitable for the experiments envisaged. The principal changes are as follows.

The measurement of axial magnetic field was automated to allow quicker adjustment of the field profile. In the present, uniform-field configuration, the axial uniformity over the central section is better than 3%. This is the best result obtainable for the present coil geometry and separation of ports in the vacuum chamber. The probe that moves along the axis of the system is now motor-driven, and its position is recorded by a potentiometer. When the probe is tracing wave patterns in the beam/plasma system, these patterns can now be registered directly on an X - Y recorder.

The work currently in progress is concerned with diagnostics. The characteristics and parameters of the plasma produced by the electron beam are being measured as a preliminary to observing wave growth and

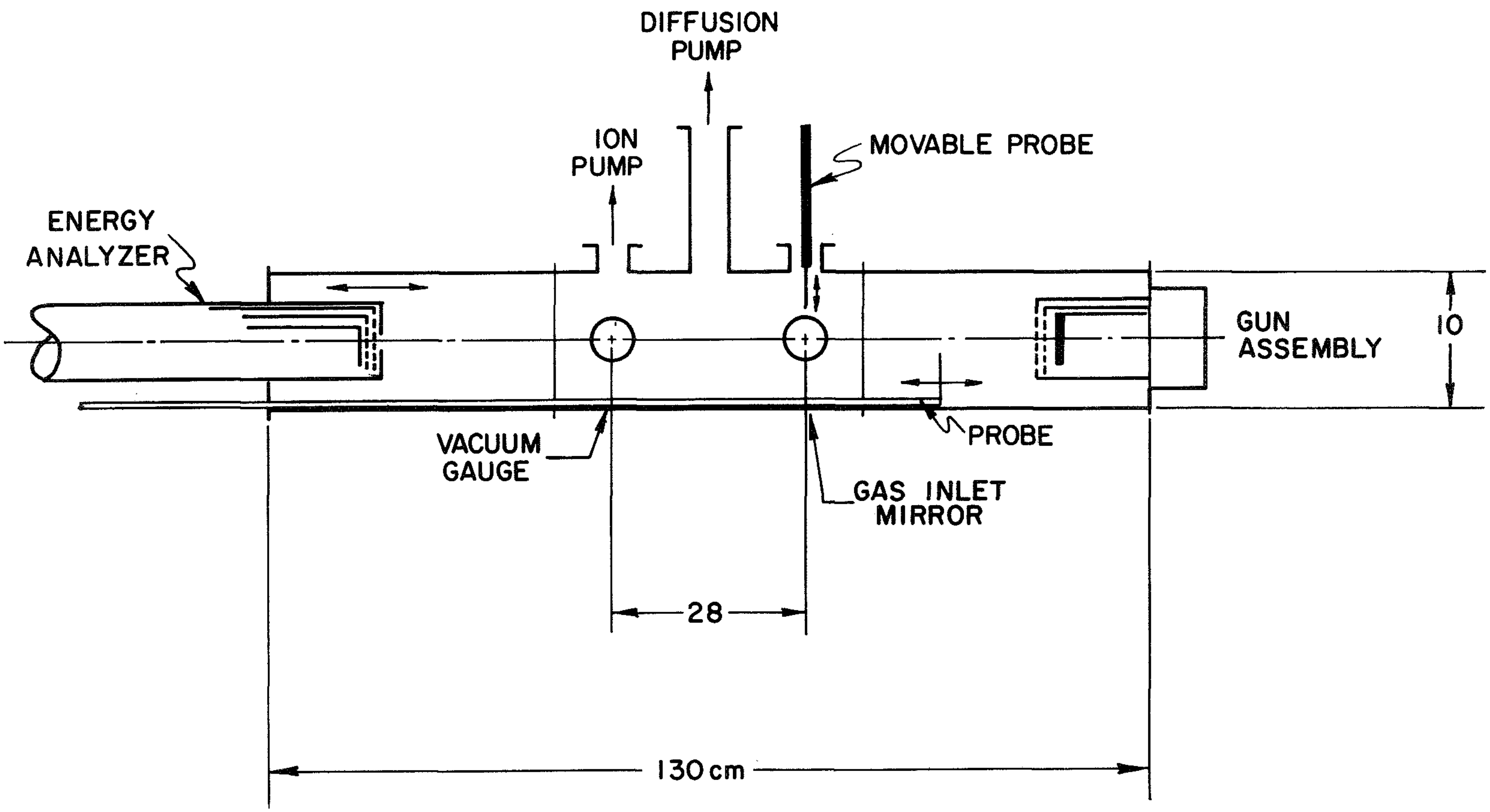

FIG. 3. SCHEMATIC OF EXPERIMENTAL SETUP FOR STUDY OF BEAM STIMULATION OF CHW.

detailed charged particle velocity distributions.

(B) Plasma Resonances in the Laboratory

Earlier under this contract we considered the excitation of CHW resonances in a coaxial plasma capacitor.[22] From the point of view of plasma diagnostics, an extremely important geometry is that of parallel wires. They would cause less perturbation of the plasma than coaxial cylinders, and lend themselves readily to construction for space applications. During the reporting period, some preliminary work has begun on extension of our previous theory to cover the parallel wire case. Details will be given in the next SAR when further progress has been made.

III. NONLINEAR SCATTERING FROM PLASMA COLUMNS

In the ionospheric literature, nonlinear mixing of signals at frequencies ω_1, ω_2 to produce $\omega_1 - \omega_2$ has already been reported.[17] Such interactions are also currently of considerable interest in laboratory plasma physics.[18] During the reporting period, we have considered the relevant problem of nonlinear scattering from a cylindrical column of plasma. This is a model approaching the ionospheric situation mentioned in Section I, i.e., that of reflection from a meteor trail. Progress is as follows:

(A) Theoretical Work

The model to be analyzed is that of an infinitely long uniform plasma column of radius r_0. We shall consider first the linear scattering, and then the nonlinear mixing.

(a) Linear Scattering. Using quasistatic theory, we know that a wave of amplitude $\hat{E}$ impinging on the column will set up fields

$$E_r = \frac{2\hat{E} \sin \theta}{1 + K} \quad , \quad E_\theta = \frac{2\hat{E} \cos \theta}{1 + K} \quad , \tag{20}$$

interior to the plasma.[23] Since there are no first-order surface currents, both H_z and E_θ are continuous across the boundary. Hence,

$$H_{zi} - H_{z0} = 0 \quad , \quad K \frac{\partial H_{z0}}{\partial r} = \frac{\partial H_{zi}}{\partial r} \quad , \tag{21}$$

and K is the effective plasma permittivity,

$$K = 1 - \frac{\omega_0^2}{\omega^2} \quad . \tag{22}$$

Outside and inside the column H_z satisfies the wave equation $(\nabla^2 + k^2)\, H_z = 0$. Therefore, outside the column we have[24]

$$H_z = \exp j\omega t \cos\theta \left[J_1\left(\frac{\omega r}{c}\right) - jY_1\left(\frac{\omega r}{c}\right)\right] + \frac{k\hat{E}}{\omega\mu_0} \exp(j\omega t - jkr\cos\theta)$$

$$\approx \left(\frac{2jAc}{\pi\omega r} - \frac{j\omega Er}{\mu_0 c^2}\right) \cos\theta \exp j\omega t \text{ (for } r \text{ small)}. \tag{23}$$

Inside, we have

$$H_z = Br\cos\theta \exp j\omega t \ . \tag{24}$$

Substituting Eqs. (25) and (24) into Eq. (21) gives

$$A = \left(\frac{\epsilon_0}{\mu_0}\right)^{1/2} \left(\frac{\omega_0 r_0}{c}\right)^2 \frac{\pi\hat{E}}{2(1+K)} \ , \tag{25}$$

and a radiated power flux of

$$P = \frac{\pi}{4}\left(\frac{\epsilon_0}{\mu_0}\right)^{1/2} \cos^2\theta \left(\frac{\omega_0 r_0}{c}\right)^3 \frac{\omega_0 r_0}{\omega r(1+K)^2} \left|\hat{E}\right|^2 . \tag{26}$$

(b) Nonlinear Scattering. Now we assume that two waves, of amplitudes $\hat{E}_\beta$ and $\hat{E}_\gamma$, impinge collinearly, and set up fields and velocities described in Eq. (22). Inside and outside the plasma, the scattered wave, E_α, chosen so that

$$\omega_\alpha = \omega_\beta + \omega_\gamma \ , \tag{27}$$

satisfies the wave equation

$$\nabla^2 H_{z\alpha} + k_\alpha^2 H_{z\alpha} = 0 \quad . \tag{28}$$

The driving force for the field is a quadrupole surface current of the form

$$J_2 = \sum_{\beta,\gamma} \rho_\beta v_{\Theta\gamma} \quad , \quad v_{0\gamma} = \eta E_{\Theta\gamma}/j\omega \quad . \tag{29}$$

Since the surface charge equals the normal field discontinuity at the surface, we obtain

$$\rho_\beta = 2\epsilon_0 \, \hat{E}_\beta \sin \Theta \, \frac{K_\beta - 1}{K_\beta + 1} \quad . \tag{30}$$

If we substitute Eqs. (27), (29) and (30) and simplify, our first boundary condition of Eq.(21) becomes

$$H_{z\alpha 0} - H_{z\alpha i} = J_2 = \frac{2j\epsilon_0 \sin 2\Theta \, \hat{E}_\beta \hat{E}_\gamma \omega_0^2 \omega_\alpha \eta}{(1+K_\beta)(1+K_\gamma)\omega_\beta^2 \omega_\gamma^2} \quad . \tag{31}$$

Our second boundary condition is obtained from one of Maxwell's equations:

$$\nabla \times \vec{H}_\alpha = j\omega_\alpha \epsilon_0 \vec{E}_\alpha + \rho_0 \vec{v}_\alpha + \Sigma \, \rho_\beta \vec{v}_\gamma \quad , \tag{32}$$

and the plasma equation

$$j\omega_\alpha \vec{v}_\alpha + \Sigma \, (\vec{v}_\beta \cdot \nabla)\vec{v}_\gamma = \eta \vec{E}_\alpha \quad . \tag{33}$$

Eliminating $\vec{v}_\alpha$, simplifying, and taking the $\Theta - \lambda$ component gives

$$-\frac{\partial H_{z\alpha}}{\partial r} = j\omega_\alpha \epsilon_0 K_\alpha E_{\alpha\theta} + \sum_{\beta,\gamma} \left[\rho_\beta v_{\gamma\theta} + \frac{j\rho_0}{\omega_\alpha}(v_\beta \cdot \nabla) v_\gamma\right] . \qquad (34)$$

Since the last term on the RHS is zero on both sides of the boundary, and also $E_{\alpha\theta}$ is continuous across the boundary, we obtain

$$\frac{\partial H_{z\alpha 0}}{\partial r} = \frac{1}{K_\alpha} \frac{\partial H_{z\alpha i}}{\partial r} . \qquad (35)$$

On the outside, H_z takes the form

$$H_{z\alpha} = A \sin 2\theta \left[J_2\left(\frac{\omega_\alpha r}{c}\right) - jY_2\left(\frac{\omega_\alpha r}{c}\right)\right] .$$

$$\approx \frac{4jA}{\pi} \frac{c^2}{\omega_\alpha^2 r^2} \sin 2\theta \text{ (for } r \text{ small)} . \qquad (36)$$

Inside, H_z has the form

$$H_z = Br^2 \sin 2\theta . \qquad (37)$$

Substituting Eqs. (36) and (37) into (31) and (35) yields

$$A = -\frac{\epsilon_0 \hat{E}_\beta \hat{E}_\gamma \omega_0^2 \omega_\alpha^3 r_0^2 \pi \eta}{2(1+K_\alpha)(1+K_\beta)(1+K_\gamma) c^2 \omega_\beta^2 \omega_\gamma^2} . \qquad (38)$$

Since the power flux is related to A and E by the relations

$$P_{\beta,\gamma} = \frac{1}{2}\left(\frac{\epsilon_0}{\mu_0}\right)^{1/2} |E_{\beta,\gamma}|^2, \quad P_\alpha = |A|^2 \sin^2 2\theta \left(\frac{\mu_0}{\epsilon_0}\right)^{1/2} \frac{c}{\pi\omega_\alpha r} \quad , \tag{39}$$

we obtain finally for the polar radiation pattern

$$P_\alpha = P_\beta P_\gamma \left(\frac{\omega_0^2 \omega_\alpha \eta}{(1+K_\alpha)(1+K_\beta)(1+K_\gamma)\omega_\beta^2\omega_\gamma^2}\right)^2 \pi\mu_0^{3/2} \epsilon_0^{1/2} \left(\frac{\omega_\alpha r_0}{c}\right)^4 \frac{c}{\omega_\alpha r} \sin^2 2\theta \quad . \tag{40}$$

The most remarkable feature of this result is that it describes a quadrupole radiation pattern in response to two dipolar exciting signals. In particular, there is no back-scattered power collinear with the exciting signals. If any were observed, it would have to be scattered from the ionosphere to the receiving point.

(B) Experimental Work

The feasibility of checking Eq. (40) in the laboratory, or in space, depends on the magnitude of the scattered power. For illumination at about 1 GHz, the factor multiplying $P_\beta P_\gamma$ on the RHS of Eq. (40) is approximately 10^{-12} in MKS units when $r = r_0$. Since

$$E_{\theta\alpha} = \frac{j}{\omega_\alpha \epsilon_0} \frac{\partial H_{z\alpha 0}}{\partial r} \quad , \tag{41}$$

the voltage on the surface of the column is given by

$$V_\alpha = - \frac{2\hat{E}_\beta \hat{E}_\gamma \eta\omega_0^2 \cos 2\theta}{(1+K_\alpha)(1+K_\beta)(1+K_\gamma)\omega_\beta^2\omega_\gamma^2} \approx 10^{-7} \hat{E}_\beta \hat{E}_\gamma \quad . \tag{42}$$

Using Eqs. (25) and (38), the ratio of the nonlinear to linear scattered powers is given by

$$\left|\frac{P_\alpha}{P}\right|^{1/2} = \left|\frac{A_\alpha}{A}\right| = |\sin\Theta| \frac{2\eta_\alpha^3}{\alpha\omega_\beta^2\omega_\gamma^2(1+K_\beta)(1+K_\gamma)} \left|\frac{\hat{E}_\beta\cdot\hat{E}_\gamma}{\hat{E}_\alpha}\right| \approx 10^{-6}\left|\frac{\hat{E}_\beta\hat{E}_\gamma}{\hat{E}_\alpha}\right| \quad . \tag{43}$$

Two experimental approaches suggest themselves: first, to suspend a plasma column in free space, and to illuminate with signals at $\omega_\beta, \omega_\gamma$. A movable horn can then be used to measure the scattered power at ω_α, and the polar pattern. Second, is the possibility of placing the tube in a microwave cavity resonant at ω_α, ω_β, and ω_γ, and measuring the power output at ω_α. We have chosen the second for our initial studies, since the cavity allows high electric fields to be applied to the plasma. For simplicity we have designed a cavity resonant in the TE_{111} and TE_{121} modes for $\omega_\alpha/2 = \omega_\beta = \omega_\gamma$. The cavity has just been completed, and preliminary measurements of nonlinear interaction are under way. Details will be given in the next SAR.

IV. FUTURE PROGRAM

During the coming reporting period, the work described here on resonant stimulation of plasma waves by means of an electron beam will be continued. It is expected that the theoretical work described in Section IIA will be completed, and that our effort will be mainly experimental. Work will continue on the two-wire antenna problem for CHW excitation and impedance determination outlined in Section IIB. If the results look promising, experiments will be undertaken to compare impedance measurements with theory.

The analysis of nonlinear resonant scattering described in Section IIIA will be extended to inhomogeneous plasma. Laboratory experiments will be extended to measure the effect, and hopefully the polar pattern of the quadrupole radiation field. The feasibility of an ionospheric experiment to verify the theory will be assessed.

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